These are your formula sheets

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Derivatives:

$$\frac{d}{dx}\, ax^n = an\, x^{n-1}$$

$$\frac{d}{dx}\, \sin ax = a \cos ax$$

$$\frac{d}{dx}\, \cos ax = -a \sin ax$$

$$\frac{d}{dx}\, e^{ax} = ae^{ax}$$

$$\frac{d}{dx}\, \ln ax = \frac{1}{x}$$

Integrals:

$$\int a\, x^n\, dx = a\frac{x^{n+1}}{n+1}$$

$$\int \frac{dx}{x} = \ln x$$

$$\int \sin ax\, dx = -\frac{1}{a} \cos ax$$

$$\int \cos ax\, dx = \frac{1}{a} \sin ax$$

$$\int e^{ax}\, dx = \frac{1}{a} e^{ax}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln\left(\sqrt{x^2 + a^2} + x\right)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a}$$

$$\int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{1}{a^2} \frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x\, dx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}}$$

Constants:

$\epsilon_0 = 8.8542 \times 10^{-12}\ \mathrm{C^2/(N\,m^2)}$

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{Wb/(A\,m)}$

$c = 2.9979 \times 10^{8}\ \mathrm{m/s}$

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Physics 208 — Formula Sheet for Final Exam

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Electromagnetic waves:

Maxwell's equations predict the existence of electromagnetic waves that propagate in vacuum with the electric and magnetic fields perpendicular and with ratio:

$$E = cB$$

The waves travel with velocity c where

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Energy in Electromagnetic waves:

The energy flow rate (power per unit area) of an electromagnetic wave is given by the Poynting vector $\vec{S}$

$$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

The magnitude of the time-averaged value of $\vec{S}$ is called the intensity of the wave

$$I = \frac{1}{2} \frac{E_{\max} B_{\max}}{\mu_0} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{1}{2} \epsilon_0 c E_{\max}^2$$

Speed of light in materials

When light propagates through a material, its speed is lower than the speed in free space space by a factor called the index of refraction

$$v = \frac{c}{n}$$

Reflection and refraction

At a smooth interface, the incident, reflected, and refracted rays and the normal to the interface all lie in a single plane. The angle of incidence and angle of reflection (measured from the normal) are equal $\theta_r = \theta_a$ and the angle of refraction is given by Snell's law:

$$n_a \sin \theta_a = n_b \sin \theta_b$$

Polarization

A polarizing filter passes waves that are linearly polarized along its polarizing axis. When polarized light of intensity $I_{\max}$ is incident on a polarizing filter used as an analyzer, the intensity I of the light transmitted depends on the angle ϕ between the polarization direction of the incident light and the polarizing axis of the analyzer:

$$I = I_{\max} \cos^2 \phi$$

Spherical Mirrors

Object and image distances:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where $f = R/2$.

Thin Lenses

Object and image distances:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

where

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Magnification

The lateral magnification for the systems described above is

$$m = \frac{y'}{y} = -\frac{s'}{s}$$

Physics 208 — Formula Sheet for Exam 3

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Forces:

The force on a charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is

$$\vec{F} = q\vec{v} \times \vec{B}$$

and the force on a differential segment $d\vec{l}$ carrying current I is

$$d\vec{F} = I d\vec{l} \times \vec{B}$$

Magnetic Flux:

Magnetic flux is defined analogously to electric flux (see formula sheet 1)

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

The magnetic flux through a closed surface seems to be zero

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Magnetic dipoles:

A current loop creates a magnetic dipole $\vec{\mu} = I\vec{A}$ where I is the current in the loop and $\vec{A}$ is a vector normal to the plane of the loop and equal to the area of the loop. The torque on a magnetic dipole in a magnetic field is

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Biot-Savart Law:

The magnetic field $d\vec{B}$ produced at point P by a differential segment $d\vec{l}$ carrying current I is

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

where $\hat{r}$ points from the segment $d\vec{l}$ to the point P.

Magnetic field produced by a moving charge:

Similarly, the magnetic field produced at a point P by a moving charge is

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\, \vec{v} \times \hat{r}}{r^2}$$

Ampère's Law: (without displacement current)

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

Faraday's Law:

The EMF produced in a closed loop depends on the change of the magnetic flux through the loop

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

When an EMF is produced by a changing magnetic flux there is an induced, nonconservative, electric field $\vec{E}$ such that

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_A \vec{B} \cdot d\vec{A}$$

Mutual Inductance:

When a changing current i_1 in circuit 1 causes a changing magnetic flux in circuit 2, and vice-versa, the induced EMF in the circuits is

$$\mathcal{E}_2 = -M\frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M\frac{di_2}{dt}$$

where M is the *mutual inductance* of the two loops

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_1}$$

where N_i is the number of loops in circuit i.

Self Inductance:

A changing current i in any circuit generates a changing magnetic field that induces an EMF in the circuit:

$$\mathcal{E} = -L\frac{di}{dt}$$

where L is the *self inductance* of the circuit

$$L = N\frac{\Phi_B}{i}$$

For example, for a solenoid of N turns, length l, area A, Ampère's law gives $B = \mu_0(N/l)i$, so the flux is $\Phi_B = \mu_0(N/l)iA$, and so

$$L = \mu_0 \frac{N^2}{l} A$$

***LR* Circuits:**

When an inductor L and a resistance R appear in a simple circuit, exponential energizing and de-energizing time dependences are found that are analogous to those found for RC-circuits. The time constant τ for energizing an LR circuit is

$$\tau = \frac{L}{R}$$

***LC* Circuits:**

When an inductor L and a capacitor C appear in a simple circuit, sinusoidal current oscillation is found with frequency f such that

$$2\pi f = \frac{1}{\sqrt{LC}}$$

Physics 208 — Formula Sheet for Exam 2

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Capacitance:

A capacitor is any pair of conductors separated by an insulating material. When the conductors have equal and opposite charges Q and the potential difference between the two conductors is V_{ab} , then the definition of the capacitance of the two conductors is

$$C = \frac{Q}{V_{ab}}$$

The energy stored in the electric field is

$$U = \frac{1}{2}CV^2$$

If the capacitor is made from parallel plates of area A separated by a distance d, where the size of the plates is much greater than d, then the capacitance is given by

$$C = \epsilon_0 A/d$$

Capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + ...$$

Capacitors in parallel:

$$C_{\text{eq}} = C_1 + C_2 + ...$$

If a dielectric material is inserted, then the capacitance increases by a factor of K where K is the dielectric constant of the material

$$C = KC_0$$

Current:

When current flows in a conductor, we define the current as the rate at which charge passes:

$$I = \frac{dQ}{dt}$$

We define the current density as the current per unit area, and can relate it to the drift velocity of charge carriers by

$$\vec{J} = nq\vec{v}_d$$

where n is the number density of charges and q is the charge of one charge carrier.

Ohm's Law and Resistance:

Ohm's Law states that a current density J in a material is proportional to the electric field E. The ratio $\rho = E/J$ is called the *resistivity* of the material. For a conductor with cylindrical cross section, with area A and length L, the *resistance* R of the conductor is

$$R = \frac{\rho L}{A}$$

A current I flowing through the resistor R produces a potential difference V given by

$$V = IR$$

Resistors in series:

$$R_{\text{eq}} = R_1 + R_2 + ...$$

Resistors in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + ...$$

Power:

The power transferred to a component in a circuit by a current I is

$$P = VI$$

where V is the potential difference across the component.

Kirchhoff's rules:

The algebraic sum of the currents into any junction must be zero:

$$\sum I = 0$$

The algebraic sum of the potential differences around any loop must be zero.

$$\sum V = 0$$

RC Circuits:

When a capacitor C is charged by a battery with EMF given by $\mathcal{E}$ in series with a resistor R, the charge on the capacitor is

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

where $t = 0$ is when the the charging starts.

When a capacitor C that is initially charged with charge Q_0 discharges through a resistor R, the charge on the capacitor is

$$q(t) = Q_0 e^{-t/RC}$$

where $t = 0$ is when the the discharging starts.

Physics 208 — Formula Sheet for Exam 1

Do NOT turn in these formula sheets!

Force on a charge:

An electric field $\vec{E}$ exerts a force $\vec{F}$ on a charge q given by:

$$\vec{F} = q\vec{E}$$

Coulomb's law:

A point charge q located at the coordinate origin gives rise to an electric field $\vec{E}$ given by

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\hat{r}$$

where r is the distance from the origin (spherical coordinate), $\hat{r}$ is the spherical unit vector, and ϵ_0 is the permittivity of free space:

$$\epsilon_0 = 8.8542 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$$

Superposition:

The principle of superposition of electric fields states that the electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges

$$\vec{E} = \sum_i \vec{E}_i$$

To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements and integrate all these elements:

$$\vec{E} = \int d\vec{E} = \int_q \frac{dq}{4\pi\epsilon_0 r^2}\,\hat{r}$$

Electric flux:

Electric flux is a measure of the "flow" of electric field through a surface. It is equal to the product of the area element and the perpendicular component of $\vec{E}$ integrated over a surface:

$$\Phi_E = \int E\cos\phi\, dA = \int \vec{E}\cdot\hat{n}\, dA = \int \vec{E}\cdot d\vec{A}$$

where ϕ is the angle from the electric field $\vec{E}$ to the surface normal $\hat{n}$.

Gauss' Law:

Gauss' law states that the total electric flux through any closed surface is determined by the charge enclosed by that surface:

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{Q_{\mathrm{encl}}}{\epsilon_0}$$

Electric conductors:

The electric field inside a conductor is zero. All excess charge on a conductor resides on the surface of that conductor.

Electric Potential:

The electric potential is defined as the potential energy per unit charge. If the electric potential at some point is V then the electric potential energy at that point is $U = qV$. The electric potential function $V(\vec{r})$ is given by the line integral:

$$V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E}\cdot d\vec{l} + V(\vec{r}_0)$$

Beware of the minus sign. This gives the potential produced by a point charge q:

$$V = \frac{q}{4\pi\epsilon_0 r}$$

for a collection of charges q_i

$$V = \sum_i \frac{q_i}{4\pi\epsilon_0 r_i}$$

and for a continuous distribution of charge

$$V = \int_q \frac{dq}{4\pi\epsilon_0 r}$$

where in each of these cases, the potential is taken to be zero infinitely far from the charges.

Field from potential:

If the electric potential function is known, the vector electric field can be derived from it:

$$E_x = -\frac{\partial V}{\partial x} \quad E_y = -\frac{\partial V}{\partial y} \quad E_z = -\frac{\partial V}{\partial z}$$

or in vector form:

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

Beware of the minus sign.